Qual Quant
DOI 10.1007/s11135-016-0453-8

Fuzzy logic programming based knowledge analysis for qualitative comparative analysis

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Abstract This paper presents a way to combine outcomes of various studies in a meta analysis framework that can use the results of mixed methods and also provide query based output for intelligent decision making. We enhance the Qualitative Comparative Analysis by utilizing fuzzy logic programming to provide effective ways of combining output of various studies. The results from various studies are used to develop an axiomatic knowledge database, and then the fuzzy programming logic engine processes it to present logical answers to decision maker's queries to the system. We present a sample application to show the methodology being applied to an example problem.

Keywords QCA · Fuzzy logic · Meta analysis

1 Introduction

Decision making, based on the results of multiple research studies in the domain of a problem, can be very difficult. The main issues related to this problem are presented below.

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Published online: 18 November 2016

1.1 Information overload

Effective decision making should be based on all the information and knowledge that is available in the domain of the problem. The problem that is faced often is that decision making is a very complex process in many cases as there can be a lot of data available, research results available as well as general knowledge about the subject available. This creates the problem of information overload (Jacoby et al. 1984; Malhotra 1982). However, it is not clear how all this information can be used to allow for optimal decision making.

1.2 Mixed research methods

Quantitative as well as qualitative research methods both lead to knowledge about the problem domain. In general, combining the benefits of both can be highly useful (Tashakkori and Teddlie 2010; Creswell 2013). Many researchers are also combining quantitative as well as quantitative and natural language data in their research (Krishen et al. 2014a, b; Stubbs 1983).

1.3 Knowledge aggregation and processing

When knowledge and data are obtained from various studies performed across disciplines and conditions, a way is needed to aggregate all the knowledge. Hence a methodology and a system is required to perform this task of aggregation. This aggregation process naturally should lead to increased knowledge and understanding of the issues. Once this knowledge is assimilated, a decision maker should be able to query the system and ask questions that can be answered by the system to the best of its aggregated knowledge.

1.4 Uncertainty in knowledge and its quantification

Each method gives results but also contains some uncertainties Motro and Smets (2012). For instance statistics based methods provide some quantification of confidence and uncertainty of the results. When multiple results are available for aggregation, then uncertainty levels of each should be used in providing overall results when knowledge queries are performed.

1.5 Knowledge based decision making

Finally a decision maker needs to make decisions in a problem domain where she has access to results of various previous research. She might have a question which has not been directly answered by previous studies directly but it might be logically implied by them. There needs to be a mechanism that is able to provide assistance to the decision maker for the specific query of such kind.

2 Background

2.1 Qualitative comparative analysis (QCA)

Qualitative comparative analysis (QCA) uses Boolean logic and specifically Karnaugh maps to combine results from various qualitative studies Ragin (2014) in order to analyze necessity and sufficiency of conditions for some statement to be true. For instance let us study the topic of distracted driving and consider three variables that could influence high levels of distracted driving. Let us consider the variables to be age, time of day, and traffic conditions. More precisely, let us consider the variables to be young driver, night time, congested traffic, and the outcome variable to be distracted driving. A truth table could be built for this relationship as shown in Table 1.

The table gives the results of various studies related to these variables. For instance the seventh row states that if the driver is young, it is night time, and the traffic is not congested, then the driver is distracted. This can be symbolized as:

$$A \wedge B \wedge \neg C \Rightarrow F \tag{1}$$

where we use the symbols from Table 2.

Using Boolean logic rules we can combine statements to simplify results. For instance, the seventh and eighth rows combined give us

$$(A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C) \Rightarrow F \tag{2}$$

Using Boolean logic rules, such as the distributive law for the logic operators, this simplifies as

$$(A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C) = A \wedge B \wedge (\neg C \vee C) = A \wedge B \tag{3}$$

Hence, we have the simplified rule

$$A \wedge B \Rightarrow F \tag{4}$$

2.1.1 Evaluation

Ragin (2008) provides two ways of evaluation of a rule: consistency and coverage.

Consistency If the antecedent of an implication rule is true, then what fraction from the data is the consequent true.

Coverage If the consequent of an implication rule is true, then what fraction from the data is the antecedent true.

By studying the data in a truth table we can also study what conditions are necessary and which conditions are sufficient for a consequent as shown in Fig. 1. In the figure, we show the symbols for the sets A and B shown with a square and a circle. The relationship $A \Rightarrow B$ indicates that $x \in A$ is sufficient for $x \in B$. In set theory, this would mean $A \subset B$.

Table 1 Distracted driving truth table

Young driver A	Night time B	Congested traffic C	Distracted driving F
False	False	False	–
False	False	True	True
False	True	False	False
False	True	True	True
True	False	False	False
True	False	True	–
True	True	False	True
True	True	True	True

Table 2 Logic symbols

Symbol	Meaning	Operator type
$\wedge$	Logic AND	Binary
$\vee$	Logic OR	Binary
$\neg$	Logic NOT	Unary

On the other hand if A is the consequent for the same drawing, then B would be necessary for A. In other words, if a consequent implies an antecedent, such as $B \Rightarrow A$ as shown, then the antecedent is necessary for the consequent. One can also have the case where the antecedent is neither necessary nor sufficient as shown in the figure.

2.2 Fuzzy set qualitative comparative analysis (fsQCA)

In the fsQCA framework the membership in a set instead of having a Boolean value, has a membership value μ, such that $\mu \in [0, 1]$. The entries of a corresponding table such as the Table 1 would look like as shown in Table 3.

The membership function for each fuzzy variable converts any value for that variable into a membership value. As an example if we have three fuzzy sets named, young, middle, and old defined for age, then different age will have different membership value for each set (see Table 2).

The logic operators have fuzzy analogs involving the membership functions. We present one possible set of t-norm and t-conorms Nguyen and Walker (2005) used in Table 4.

Relationships like $A \Rightarrow B$ can be performed by transforming them using the basic operators, such as $A \Rightarrow B$ is the same as $\neg A \vee B$. Set relationship $A \subset B$ in fuzzy logic is equivalent to saying that $\mu(A) \leq \mu(B)$. The evaluation of necessity and sufficiency of conditions can also be performed by studying the fraction of instances when the fuzzy inequality between two sets is satisfied. (Figs.2 and 3).

As a concrete example if we want to evaluate the rule $A \wedge B \wedge \neg C \Rightarrow F$ using the first row of Table 3, we would obtain

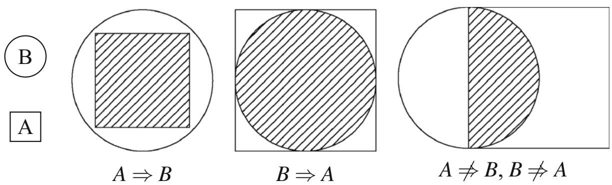

Fig. 1 Necessity and sufficiency for conditions

Table 3 Distracted driving truth table

Young driver A	Night time B	Congested traffic C	Distracted driving F
0.2	0.1	0.3	0.4
0.4	0.7	0.9	0.8

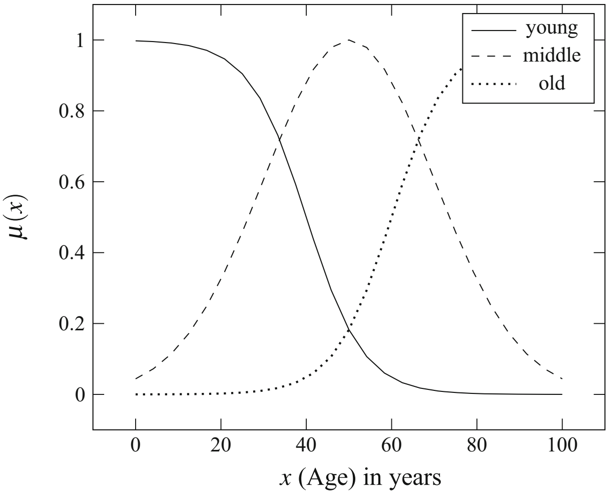

Fig. 2 Fuzzy membership functions

Table 4 Logic symbols

Symbol	Meaning	Operator	Relationship
$\wedge$	Logic AND	$A \wedge B$	$\mu(A \wedge B) = \min(\mu(A), \mu(B))$
$\vee$	Logic OR	$A \vee B$	$\mu(A \vee B) = \max(\mu(A), \mu(B))$
$\neg$	Logic NOT	$\neg A$	$\mu(\neg A) = 1 - \mu(A)$

$$\mu(A \wedge B \wedge \neg C \Rightarrow F) = \mu(\neg(A \wedge B \wedge \neg C) \vee F) \tag{5}$$

Applying DeMorgan's law $\neg(\wedge_{i \in \omega} A_i) = (\vee_{i \in \omega} \neg A_i)$ in equation (5), we get

$$\begin{aligned} \mu(\neg(A \wedge B \wedge \neg C) \vee F) &= \mu(\neg A \vee \neg B \vee C \vee F) \\ &= \max(\mu(\neg A), \mu(\neg B), \mu(C), \mu(F)) \\ &= \max((1 - \mu(A)), (1 - \mu(B)), \mu(C), \mu(F)) \\ &= \max(1 - 0.2, 1 - 0.1, 0.3, 0.4) = \max(0.8, 10.9, 0.3, 0.4) = 0.9 \end{aligned} \tag{6}$$

3 Decision support system

When the results of multiple studies on a related topic are obtained, there are many ways a researcher can go about aggregating the output and make intelligent decisions in the problem domain. We have already seen that using QCA and fsQCA we can combine the results to come up with new aggregated and simplified statements.

3.1 Inverse problem in decision support system

Many policy makers and decision makers need to make a decision on a policy or project. The decision should come using various studies and past data. In general it is a difficult problem to make sure that a decision is consistent with all the knowledge gained from multiple studies. This paper addresses this exact problem by enhancing the fsQCA for this purpose.

3.2 Overall methodology

The basic structure of our framework that addresses this issue is composed of a knowledge database created from results of various studies combined with a set of inference rules that allow a user to query the system to obtain logical responses from the developed system. The structure of the framework has two parts, the construction phase, and the deployment phase. In the construction phase we build the knowledge database and the inference rules based on the outcomes of various studies, and then in the deployment phase we can query the system with logical questions we want answered and the system can respond with either letting the user know if the logical reply to the query is true or false, or if there is insufficient knowledge in the system to answer that question.

4 Boolean logic based

Boolean logic based system to combine results from multiple studies and allow for logical queries to be made can be built around the Prolog programming language Malpas (1986). A Prolog program has a knowledge part and a query interface. The knowledge part has two components: (a) Basic facts (unit clauses) such as s_1, s_2 etc., which mean s_1 is true, s_2 is true etc. (b) Inference rules in terms of conditional clauses which help to deduce further knowledge, such as $s_1 : -s_2, s_3$, which means if s_2 and s_3 are true, then s_1 is true. The query interface is the user-interface mechanism which allows the user to query the system and then obtain the result of analysis back.

The inference engine automatically uses various rules in an algorithm to find out if the query system is true or not. The following rules of inference are some of the rules that are normally used.

Modus Ponens: $((P \Rightarrow Q) \wedge P) \Rightarrow Q$
Modus Tollens: $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow P$
Hypothetical Syllogism: $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
Disjunctive Syllogism: $((P \vee Q) \wedge \neg Q) \Rightarrow P$

4.1 Case study

In our case study, we will show how to perform meta analysis and knowledge exploration based on two different studies on a similar topic using different methods. The first study is Krishen et al. (2016a) and the second one is Krishen et al. (2016b).

The following are the hypotheses from the paper "Is having accurate knowledge necessary for implementing safe practices? A consumer folk theories-of-mind perspective on the impact of price" Krishen et al. (2016a).

H1 Perceived price associates negatively with product experience.

H2 Product experience associates positively with attitude toward product.

H3 Product experience associates positively with actual knowledge.

H4 Attitude toward the product associates positively with reported use of the product.

H5 The product actual knowledge associates positively with reported use of the product.

Table 5 provides the constructs from the paper "Framing the value and valuing the frame? Algorithms for child safety seat use" Krishen et al. (2016b).

The rules obtained by applying the fsQCA technique were:

R1 Intention to Use = Risk Aversion ∧ Perceived Knowledge ∧ Attitude towards object ∧ Product Experience ∧ Low Price Conscious

R2 Intention to Use = Low Price Attitude ∧ Attitude towards SS ∧ Perceived Knowledge ∧ Attitude towards object ∧ Product Experience

R3 Intention to Use = Price Attitude ∧ Risk Aversion ∧ Perceived Knowledge ∧ Attitude towards object ∧ Product Experience

To develop our Prolog knowledge system, we use the online Prolog at http://swish.swi-prolog.org/. We develop the knowledge base by converting the hypotheses into logical propositions, as shown in Listing 1.

```
% Knowledge base

associates_negatively_with(perceived_price, product_experience).
associates_positively_with(product_experience, attitude_toward_product).
associates_positively_with(product_experience, actual_knowledge).
associates_positively_with(attitude_toward_product, reported_product_use).
associates_positively_with(actual_knowledge, reported_product_use).

associates_positively_with(X, Z) :-
    associates_positively_with(X, Y),
    associates_positively_with(Y, Z).

associates_positively_with(X, Z) :-
    associates_negatively_with(X, Y),
    associates_negatively_with(Y, Z).
```

Listing 1: Prolog Knowledge Base built from (Krishen et al., 2016a)

As an example, the hypothesis "Perceived price associates negatively with product experience" is translated into a proposition of propositional logic as `associates_negatively_with(perceived_price, product_experience)`.

The transitivity of the relation `associates_positively_with(X,Y)` is developed in the code:

```
associates_positively_with(X, Z) :-
    associates_positively_with(X, Y),
    associates_positively_with(Y, Z).
```

Similarly, the relationship between the mathematical relations `associates_positively_with` and `associates_negatively_with` is developed in the code:

Table 5 Framing constructs

Construct Items	
RISKAVERCOMBINED	
1.	I would rather be safe than sorry
2.	I want to be sure before I purchase anything
3.	I avoid risky things. Donthu and Garcia (1999)
4.	I don't like to take risks
5.	Compared to most people I know, I like to live life "on the edge"
6.	I have no desire to take unnecessary chances on anything.
7.	Compared to most people I know, I like to gamble on things. Burton et al. (1998)
PRICEATT Yoo et al. (2000)	
1.	The price of child safety seat is high
2.	The price of child safety seat is low
3.	Child safety seats are expensive
PERCEIVEDKNOW Beatty and Smith (1987)	
1.	I have a lot of experience with child safety seats
2.	As compared to the average person, I would say that I am highly knowledgeable about the child safety seat product category
3.	I would describe myself as being very familiar with the child safety seat product category
ATT_SS	
1.	Child safety seats are too expensive to be used
2.	To prevent injury during a crash it is always safer to use a child safety seat
3.	It is more convenient for a passenger to hold a child
4.	Safety seats must be changed as a child grows
5.	It is important to use a child safety seat for a small duration trip
6.	To prevent severe injury it is always better to use a safety seat.
PRODUCTEXPR Griffin et al. (1996)	
1.	I have a great deal of skill in using child safety seats
2.	I make use of child safety seats frequently
3.	I have experience using child safety seats
4.	I know how to operate child safety seats
ATT_OBJECT Lord et al. (1994)	
1.	Child safety seats are a good idea
2.	Child safety seats are a favorable idea
3.	Child safety seats are a pleasant idea
INTENTIONTOUSE	
1.	How often do you use a child safety seat while driving with children under the age of 14 years?
2.	How often do you use a child safety seat?
3.	How regularly do you use a child safety seat?
PRICECONSCIOUS Shim and Gehrt (1996)	
1.	I buy as much as possible at sale prices
2.	The lower price products are usually my choice
3.	I look carefully to find the best value for the money

```
associates_positively_with(X, Z) :-
    associates_negatively_with(X, Y),
    associates_negatively_with(Y, Z).
```

Once the knowledge base is built, the researcher can query the system with questions to check their validity. One example of a query is:

```
?- associates_positively_with(product_experience, reported_product_use).
```

The system responds with `true` as using the transitivity property and hypothesis gives us the result. Mathematically, if our query is the proposition P, the proof is $H2 \wedge H4 \Rightarrow P$. Although this query and the proof are simple ones, but we can build a very large knowledge base and then many complex queries can be asked to the system.

In order to perform meta-analysis, we can add propositions from each of the different studies. For our case study, we can convert the results R1, R2, and R3 also into logical propositions inside our knowledge database. For instance the results R1 can be programmed as:

Risk Aversion $\wedge$ Perceived Knowledge $\wedge$ Attitude towards object $\wedge$ Product Experience $\wedge$ Low Price Conscious $\Rightarrow$ Intention to Use.

Combining the new rules with the previously developed database provides a bigger knowledge bank which the user can query. For instance, if we know that Risk Aversion , Perceived Knowledge, Attitude towards object, and Low Price Conscious are all true propositions, then the Intention to use will also be true because of the proposition related to H2.

5 Fuzzy logic based propositional logic

Fuzzy logic based system to combine results from multiple studies and allow for logical queries to be made can be built around a fuzzy Prolog based programming language Mukaidono et al. (1989). In the crisp knowledge database with the Prolog query system, the analysis followed the law of the excluded middle, and hence there was no way to use the uncertainties in knowledge from different studies. Different types of studies (quantitative as well as qualitative) have different methodologies to quantify uncertainties. For instance in our case study the hypotheses relationships have β strength to show uncertainty. Similarly in fsQCA we have consistency and coverage values. Fuzzy Prolog can be built to represent fuzzy knowledge database and then perform queries on it. various versions of fuzzy Prolog have been developed by researchers (Li and Liu 1990; Guadarrama et al. 2004; Martin et al. 1987; Vaucheret et al. 2002) and some implementations are freely available online as of the date of this writing.

There are many different ways to incorporate partial truth in logic, and consequently in logic programming. Some systems use the resolution principle (Lee, 1972), and some use the semiring based min-max system. Three valued, or multi valued logic has also been used to build fuzzy propositional logic systems Klawonn and Kruse (1994). Interval based constraint logic programming is used in Prolog developed by Vaucheret et al. (2002).

5.1 Fuzzy language and query system

Following Vaucheret et al. (2002) we define the following:

Definition 1 (*Term*)

1. Any constant is a term
2. Any variable is a term
3. $f(t_1, t_2, \cdots, t_n)$ where t_i, $i = 1, 2, \cdots, n$ are terms and f is a function, is a term

Definition 2 (*Atomic formula* (*Atom*)) $P(t_1, t_2, \cdots, t_n)$ where t_i, $i = 1, 2, \cdots, n$ are terms and P is a predicate, is an atomic formula (also called atom in short).

Definition 3 (*Fuzzy fact*). $v \rightarrow A$ where v is a fuzzy value and A is an atom, is a fuzzy fact.

The truth value depends on what specific type of logic is being used. If the logic is binary then any proposition/predicate has either the *true* value $\top$ or *false* value $\bot$. A three valued logic can use values *true* $\top$, *false* $\bot$, and uncertain U. We can of course use multi finite valued logic, or a continuous logic such as the interval based.

Definition 4 (*Fuzzy clause*) $(P_1 \wedge P_2 \cdots P_n) \rightarrow_{\mathfrak{a}} A$ where P_i, $i = 1, 2, \cdots, n$ and A are atoms, and $\rightarrow_{\mathfrak{a}}$ is an aggregator (such as a min function or some t-norm), is a fuzzy clause.

Definition 5 (*Fuzzy query*) $?A \rightarrow v$ where A is an atom and v is a variable with truth value, is a fuzzy query.

Definition 6 (*Fuzzy knowledge base*) Any finite set of fuzzy facts and fuzzy clauses that we can query is called a fuzzy knowledge base.

5.2 Case study for fuzzy meta analysis

In this subsection we show how to perform meta analysis with uncertainties from multiple studies using different types of methods (quantitative as well as qualitative). For simplicity we will use Łukasiewicz three valued logic, L_3. We will use the same studies as used in subsect. 4.1. Each proposition obtained from each study would have to be given a truth value, and in the case of three valued logic, the value v would satisfy $v \in \{\bot, \mathrm{U}, \top\}$.

As shown in Fig. 4, fuzzifiers are used to transform the hypotheses and rules from each study into fuzzy clauses of the fuzzy database. The exact technique depends on what type of study each one is. For instance the β values from study 1 can be used to decide the truth value for each hypotheses. Similarly, the consistency and coverage values for rules can be used in fsQCA based studies to decide the truth values of clauses.

The logical processing of the query input with the knowledge base uses Łukasiewicz three valued logic that we have chosen. That logic follows the rules given in Table 6. We use 0 instead of $\bot$, and 1 instead of $\top$, and retain U for the third truth value for the table.

For comparison purpose we will now use the same query as was used for crisp set case and see how the fuzzy system resolves this query. The query again is:

```
?- associates_positively_with(product_experience, reported_product_use).
```

Let us use the truth value for H2 hypotheses as $\top$ and that for H4 as U based on their β values from their original study analysis (see Fig. 3). The system responds with U as using

Table 6 Łukasiewicz L_3 Logic

x	y	∧	∨	⇒	⇔
0	0	0	0	1	1
0	U	0	U	1	U
0	1	0	1	1	0
U	0	0	U	U	U
U	U	U	U	1	1
U	1	U	1	1	U
1	0	0	1	0	0
1	U	U	1	U	U
1	1	1	1	1	1

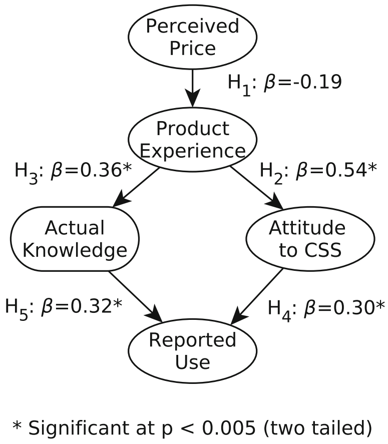

Fig. 3 Theme and concept connections

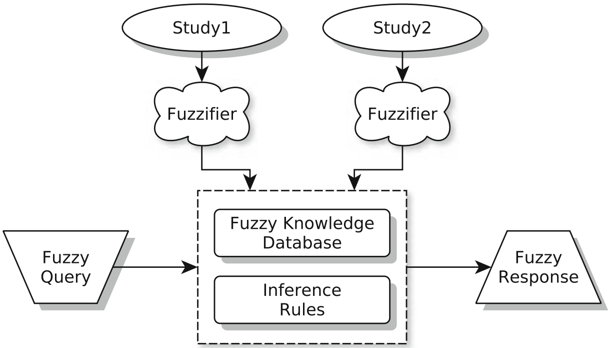

Fig. 4 Fuzzy framework structure

the transitivity property and hypothesis gives us the result. Mathematically, if our query is the proposition P, the proof is $H2 \wedge H4 \Rightarrow P$ where we use the 8*th* row of Table 6.

6 Conclusion and business implications for managers

In this paper we have shown how meta analysis of a topic can be performed by combining results from different studies that can involve various quantitative and qualitative techniques. The methodology was based on using propositional logic and then enhancing it by using a fuzzy version of the same. It was shown how the Prolog programming language can be used for the task in the case without uncertainties, and fuzzy Prolog for the case with uncertainties. Software tools are freely and easily available that make the task for researchers to perform meta analysis easy.

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